Semi-Empirical Models for the Variation of Soil Complex Permittivity with Depth

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Abstract:

In this paper new semi-empirical formulas are developed to evaluate the variation of both real and imaginary parts of soil complex permittivity with depth inside the earth's surface. Computed values using these models show good agreement with published measured values for soils of the same textures and same frequency band. Use of these models may serve to handle more accurate results especially in the ground probing radar (GPR) applications and other applications relating the detection of buried objects inside the earth's surface, where the use of a single average value of the soil complex permittivity had not necessarily led, for most of the times, to accurate results for the electromagnetic fields propagated inside the earth's surface.

صيغتان شبه ريبية لتغ السماحية ا ركبة للتربة مع العمق

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ا لاصة: يقدم هذا البحث صيغة شبه ريبية جديدة لتقييم التغ ات ا اصلة في ا زئ ا قيقي وا يا للسماحية ا ركبة للتربة مع العمق ت سطح الأرض. اظهرت القيم ا سوبة بأستخدام هاتا الصيغة التجريبية تطابقا جيدا مع تلك ال جرى قياسها ونشرها الادبيات العلمية ذات العلاقة ت الظروف نفسها من نوع التربة وتركيبها والنطاق الترددي الذي القياس خلاله. ان استخدام الصيغة التجريبية ا عروضة هذا البحث سوف دم ا صول على نتائج اكثر دقة خصوصا تطبيقات رادرات س الأعماق و تطبيقات كشف الأجسام ا دفونة ت سطح الأرض حيث ان الأعتماد على قيمة متوسطة واحدة للسماحية ا ركبة للتربة للآعماق ا ختلفة لايؤدي ا نتائج دقيقة اغلب ا الات للمجالات الكهرومغناطيسية ا نتشرة داخل سطح الأرض.

1-Introduction:

Soil complex permittivity, among many other factors, is very important in the evaluation of reflected, scattered, and transmitted electromagnetic fields inside the earth's surface at different frequency bands. In studying penetrated fields inside the earth surface this requires an accurate knowledge of both the real and the imaginary parts of the soil complex permittivities at different depths. This seems to be of great importance in the study of detection of buried objects inside the earth surface and in the ground probing radar (GPR) applications. In most cases there is a problem arising from the existence of moisture distributed through the different depths.

In spite of the relatively large measurements of complex permittivity of moist soils at different frequency bands, no attempt, as the authors believe, had been made to model its behavior with depth. Extensive experimental works had been performed to measure the complex permittivity of soils of various types at different frequency bands. Some of these measurements were presented in the form of tables with no correlation among the results obtained [1,2], while others had presented their results as empirical formulas describing the behavior of both real and imaginary parts of soil complex permittivity with moisture and soil texture at certain frequency bands[3,4,5,6].

The general form of these formulas is a second or third order polynomial with the moisture as the independent variable and constants depending on soil texture and frequency band.

Recent trends in this field of research are to measure the penetration of electromagnetic signals at different frequencies for different depths inside the earth's surface [7,8].The penetration depth is [9] defined as "the distance in the medium over which the intensity of propagating radiation decreases (owing to attenuation) by the exponential factor e-1 (i.e., by about 63%)." However, because of the large contrast between the permittivities of dry soil and water, and because the amount of water in the soil is variable, soil moisture largely controls the permittivity of the soil and thus penetration depth as well. Analyzing the measured data, information could be obtained about soil permittivity at different depths inside ground surface.

2-Formulation of the Problem

The developed model is based essentially on two empirical formulas; the first one had been developed to present the dependence of moisture content in the soil with depth away from the ground surface when it is irrigated [10]. It describes the variation of moisture at different depths when the ground is subjected to several irrigation cycles. Also this formula describes this variation with time after irrigation. According to this model, the variation of water content, in various depths, obeys the relation:

$$w = 0.256\, D\, T^{(-0.128)} \qquad (1)$$

where w is the volumetric water content, D is the depth in cm, and T is the time in days after cessation of irrigation.

The second empirical formula describes the behavior of soil relative permittivity with its moisture. It is in the form [5,6]:

$$\varepsilon_r' = A + Bw + Ew^2 \qquad (2)$$

$$\varepsilon_r'' = A' + B' w + E' w^2 \qquad (3)$$

and

where ε_r' *and* ε_r'' are the real and imaginary parts of the soil complex permittivity, and the factors *A, B, E, A', B'*, and *E'* are constants, their values depend on the soil type and the operating frequency. Values of these constants, at certain frequency band, depend on the soil texture. Their values depend primarily on the sand and clay contents of the examined soil.

Knowing the soil texture, the constants *A*, *B*, and *E* can be found in the form [6]:

$$\begin{bmatrix} A \\ B \\ E \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ S \\ C \end{bmatrix} \qquad (4)$$

where S and C are the sand and clay contents in the soil, while the elements a_{mn} can be extracted from tables at certain frequency band [11]. Similarly *A, B*, and *E* can be found with their corresponding matrix [a'_{mn}].

From these two formulas, a model describing the behavior of soil relative permittivity with depth away from ground surface will be estimated. Empirical formulas describing the dependence of the real part of soil complex permittivity (ε_r) on depth (*D*) under ground surface and the time (*T*) after irrigation are found to be:

$$\varepsilon_r' = A + 0.256 B D^2 T^{(-0.128)} + 0.065 E D^2 T^{(-0.256)} \qquad (5)$$

and

$$\varepsilon_r'' = A' + 0.256 B' D T^{(-0.128)} + 0.065 E' D^2 T^{(-0.256)} \qquad (6)$$

There are no serious limitations on the use of the two models, except what is called the soil field capacity (w_{fc}) which represents the value of soil moisture content that can be handled by the soil such that any excessive moisture caused it to be saturated [10]. Soil field capacity depends extremely on the soil texture, and an empirical model had been developed by one of the authors [12]. Hence all computations using the developed models must deal with situations where soils having moisture contents below (w_{fc}). For situations with soils having moisture contents equal or exceed (w_{fc}) these models are not valid.

3-Sample of Results

Both real and imaginary parts have been evaluated for soils with any texture and for different frequency bands, as a function of depth inside the earth's surface. Therefore a wide variety of results can be expected to be obtained.

A sample from these results is shown in Figs. (1 to 5). The soil type chosen is the sandy clay with a texture of 46% clay, 29% silt and 25% clay and the test frequency is 10 GHz.

At this frequency:

and

$$[a_{mn}] = \begin{bmatrix} 2.5 & -0.003 & -0.003 \\ 10.1 & 0.22 & -0.004 \\ 77.5 & -0.061 & 0.135 \end{bmatrix} \qquad (7)$$

$$[a'_{mn}] = \begin{bmatrix} -0.07 & 0 & 0.001 \\ 6.62 & 0.015 & -0.081 \\ 21.59 & 0.291 & 0.33 \end{bmatrix} \qquad (8)$$

Use of these values together with the variation of water contents with depth and time, the real and imaginary parts of this soil sample can be evaluated.

Fig.(1) shows the dependence of moisture contents on time ,for three different depths. Figs(2) and (3) show the dependence of the real and the imaginary parts of the soil permittivity on time after irrigation, while Figs.(4) and (5) demonstrate the variation of the real and imaginary parts of the permittivity with depth for different times. Results presented in Figs (1-3) have been found to be in good agreement with those presented in [7,8] after appropriate conversion for same soil texture, frequency band and depths inside the earth's surface. These measured results have been presented in a graphical form of depth of penetration versus depth inside the earth's surface. Depth of penetration can be considered as a measure of soil complex permittivity and vice versa. A convenient expression for penetration depth as a function of permittivity (using the small angle approximation) is [13]:

$$\delta_p = \frac{\lambda\sqrt{\varepsilon_r^{'}}}{2\pi\varepsilon_r^{"}} \qquad (9)$$

where δ_p is penetration depth and λ is wavelength, both in millimeters. This expression has been used for results conversion to show the validity of the proposed models.

4-Dicussion of the Results

Figure (6) demonstrates the results offered by the proposed models, together with those reported in [8], for the sake of comparison. Calculation of the depth of penetration δ_p have been based on Equ.(9) and the use of Equs.(5) and (6).

The general behavior of the moisture profile and δ_p with time resulting from the proposed models well agree with reported in [8]. The slight difference shown in Fig.(6) might be explained as follows. The soil sample used in [8] may be texturally different from that used in the present work. The most important factor that the water handling capability of a soil depends on is the sand content. The lower the sand content of a soil, the higher its water handling is.

As mentioned earlier, the soil sample used in this work has a sand content of x%. This may be mainly responsible of the lower moisture content noticeable in the proposed model results. The second reason, which is of less importance, is the frequency according to which computations have been carried out. In the present work, the computations are performed at a frequency of 10 GHz, which is the center of the X-band (8-12 GHz). In [8], it has been reported that the measurements were roughly performed in the X-band. As a matter of fact, at the upper X-band frequencies, the depths of penetrations are slightly lower than those obtained at the lower frequencies.

In conclusion, it is likely that in [8], a soil sample with lower sand content has been used and the measurements have been carried out at an upper X-band frequency , are the most probable reasons for the differences in Fig.(6).

5-Conclusion

New semi-empirical models had been developed for the estimation of soil complex permittivity and its dependence on depth inside the earth's surface and time after the surface is being irrigated. It is useful for identifying any material in the vicinity of the moist soil; which is of importance in the detection of buried objects.

All the displayed results are given for a defined soil texture and an operating frequency of 10 GHz. Families of curves can be obtained at other frequency bands or for different types of soils using the developed models. Results obtained using the proposed models show good agreement with measured practically for same soil types and at the same frequency band.

Use of these models present a simple means for evaluating both real and imaginary parts of the relative complex permittivity for a wide variety of soils at different frequency bands as functions of depth inside the earth's surface with consumed both time and effort compared with the costly and time consuming practical in situ measurements.

References

[1] R.L. Smith-Rose, "The Electrical Properties of Soils for Alternating Currents at Radio Frequencies," *Proc. Roy. Soc. Serv. A, 140,* pp. 221-235, 1935.

[2]. W. P. Waite, "Microwave Bistatic Reflectivity Dependence on Moisture Contents and Matric Potential of Bare Soil," *IEEE, Trans. Geosci. Remote Sensing*, vol. GE-22, no. 4, July 1984.

[3]. F. E. Geiger and D. Williams, "Dielectric Constant of Soils at Microwave Frequencies, *NASA-TM-657987*, April, 1972.

[4]. P. Hoekstra and A. Delaney, "Dielectric Properties of Soils at Microwave Frequencies," *.J. Geophys. Res.* vol. 79, pp. 1699-1709, 1974.

[5]. J. R. Wang, "Dielectric Properties of Soils-Water Mixtures at Microwave Frequencies," *NASA-TM-80597,* March 1980.

[6]. M. C. Dobson, "Microwave Dielectric Behavior of Wet Soil," *IEEE, Trans. Geosci. and Remote Sensing*, vol. GE-23, no. 1, Jan. 1985

[7]. Kyeonghwan .H. Lee, "A Dielectric Permittivity Sensor for Simultaneous Measurement of Multiple Soil Properties," Ph.D Thesis, *Kansas State University*, May, 2006.

[8] Matt Nolan and Dennis R. Fatland, "Penetration Depth as a DInSAR Observable and Proxy for Soil Moisture," *IEEE Trans. on Geosci, and Remote Sensing*, vol. 41, no. 3, pp. 532-537, March 2003.

[9] E. G. Njoku and D. Entekhabi, "Passive Microwave Remote Sensing of Soil Moisture," *J. Hydrol.*, pp. 101–129, 1996.

[10]. W. A. Judy, W. H. Gardner and W. R. Gardner, "Soil Physics," *Wiley, New York,* 1991.

[11].Jawad K.Ali, Jabir.S.Aziz, and Mahmoud T.Al-Naimy, Complex Permittivity Measurement of Soil in the Middle and South of Iraq at 3 cm Wavelength,*4th Scientific Conference , Scientific Research Council*, 23-28, Oct., 1986, *Baghdad, Iraq*.

[12]. Jawad K.Ali, Jabir.S.Aziz, and Mahmoud T.Al-Naimy, "Soil Saturation and Soil Field Capacity: Empirical Formulas," *IEEE-MELECON,* March 1987. *Rome Italy.*

[13] F. T. Ulaby, R. K. Moore, and A. K. Fung, Microwave Remote Sensing, *Reading, MA:* vol. 2, Fundamentals of Radiometry. *Addison-Wesley*, 1982.

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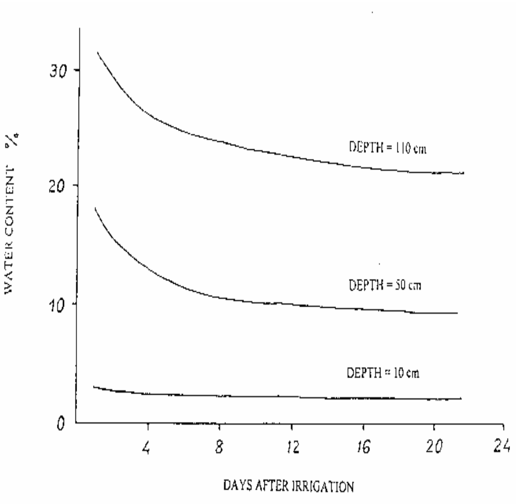

Fig.1. Variation of water content with time after irrigation. .

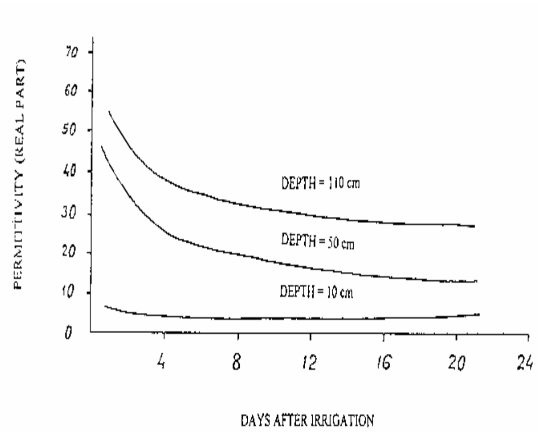

Fig.2. Dependence of soil relative permittivity (real part) on time after irrigation.

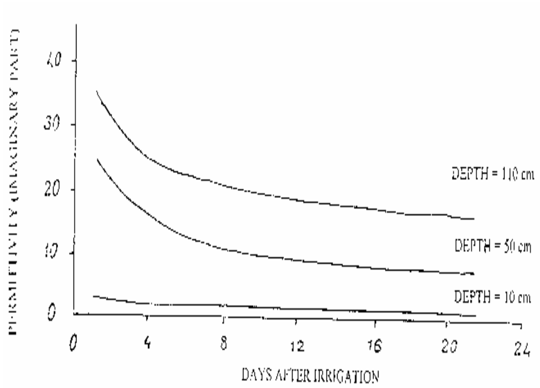

Fig.3. Dependence of soil relative permittivity (imaginary part) on time after irrigation.

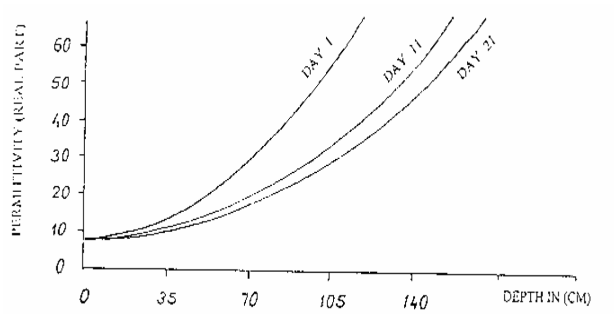

Fig.4. Dependence of soil relative permittivity (real part) on depth inside ground surface.

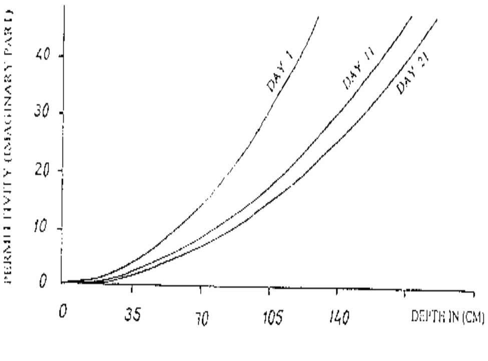

Fig.5. Dependence of soil relative permittivity (imaginary part) on depth inside ground surface.

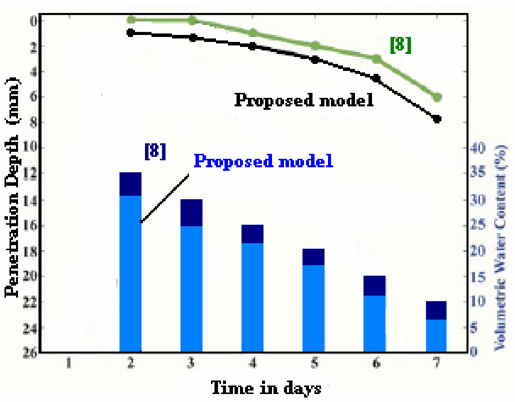

Fig.6. Penetration depth and the water content profiles resulting from the proposed models together with those reported in [8].