Proposed Method for Optimizing Fuzzy linear programming Problems by using Two-Phase Technique

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Abstract

Fuzzy linear programming (FLP) is an application of fuzzy set theory in linear decision making problems and most of these problems are related to linear programming contains fuzzy constrains or crisp objectives function or contains crisp constrains with fuzzy objectives function, which called fuzzy linear programming (FLP) with triplet fuzzy numbers consist a hybrid fuzzy. The crisp constrains used in the problems of types (= or ≥) with a proposed optimization fuzzy objectives and fuzzy constrains. In this paper proposed method for solving fuzzy linear programming problem by using Two-phase technique to solve the problem and to determine the optima crisp objectives.

Keywords: Linear Fuzzy Real Numbers, Fuzzy Linear programming
Approach, Fuzzy constraints.

1. Introduction

Linear programming is one of the most important operational research (OR) techniques. It has been applied to solve many real world problems but it fails to deals with imprecise data. The concept of a fuzzy decision making was first proposed by Bellman and Zadeh, 1970. Recently, much attention has been focused on FLPP (Campose and Verdegay, 1989; Mahmoud and Abo-Sinna, 2004; Negoita, 1970; Takashi, 2001; Takeshi et al., 1991) (Zimmermann, 1978). [1].

Decision making is possibly the most important and inevitable aspect of application of mathematical methods in various fields of human activity. In real-world situations, decisions are fuzzy, at least partly. The first step of attempting a practical decision-making problem consists of formulating a suitable mathematical model of a system or a situation to be analyzed.

Linear programming (LP) is a mathematical modeling technique designed to optimize the usage of limited resources. The determination of optimal solution for making decision that helps to infer that by incorporating fuzziness in a linear

programming model either in constraints, or both in objective functions and constraints, provides a similar (or even better) level of satisfaction for obtained results compared to non-fuzzy linear programming with crisp objectives and fuzzy constraints, or the fuzzy multi objective decision model, with both fuzzy objectives and fuzzy constraints[2].

Fuzzy Linear Programming problem associates fuzzy input data by fuzzy membership functions. Fuzzy Linear Programming model assumes that objectives and constraints in an imprecise and uncertain situation can be represented by fuzzy sets. The fuzzy objective function can be maximized or minimized. In Fuzzy Linear Programming the fuzziness of available resources is characterized by the membership function .

In this paper, the theory of a tripartite interactive Fuzzy Linear Programming (FLP) is proposed and an application of this proposal in solving for a profit function in an industrial production planning problem is suggested. The tri partite FLP gets required data inputs from the analyst, the decision maker and the implementer. The data and he constraints are fuzzy in their characteristics. The entire problem of profit optimizations solved in an interactive way among the analyst, decision maker and the implementer. In the tripartite planning, the author is the analyst who gets data from the other two and searches for a solution in an effort to satisfy them. Now a days, it is almost impossible to achieve a successful development without these three participants in the interactive decision making process.

We denote this hybrid set of numbers as Linear Fuzzy Real numbers (LFR). The set of LFR is a set that shows true intermediate properties which are unique to the set and not to those of either the real numbers or the "general" fuzzy numbers. Because of the unique properties of LFR, we can solve fuzzy linear optimization problems[3,4,5].

The outlined in this paper as follows. Linear Fuzzy Real Numbers with triple of real numbers (*a, b, c*). we study Fuzzy Linear programming with Algorithm of fuzzy linear programming by using Two-phase. We are giving two examples to explain the operations of the linear fuzzy programming using LFR. And discuss the applications of LFR optimization.

2. Linear Fuzzy Real Numbers

Considering the real numbers R, one way to associate a fuzzy number with a fuzzy subset of real numbers is as a function $\mu : R \rightarrow [0, 1]$, where the value $\mu(x)$ is to represent a degree of belonging to the subset of R. The Linear Fuzzy Real numbers as described by Neggers and Kim [5, 3] is a triple of real numbers (a, b, c) where $a \leq b \leq c$ of real numbers, See Fig. 1, such that:

1. $\mu(x) = 1$ if $x = b$;
2. $\mu(x) = 0$ if $x \leq a$ or $x \geq c$;
3. $\mu(x) = (x - a)/(b - a)$ if $a < x < b$;
4. $\mu(x) = (c - x)/(c - b)$ if $b < x < c$.

For a real number c, we let $€(c) = \mu$ with associated triple (a, b, c). Then μ is a linear fuzzy real number with $\mu(c) = 1$ and $\mu(x) = 0$ otherwise. As a linear fuzzy real number we consider $€(c) = \mu$ to represent the real number c itself. Thus by this interpretation we note that the set R of all real numbers is a subset of the set containing the linear fuzzy real numbers. The set of the linear fuzzy

real numbers is a hybrid set showing intermediate properties, which are unique to the set and not those of either the real numbers or the "general" fuzzy numbers, show in Figure (1) .

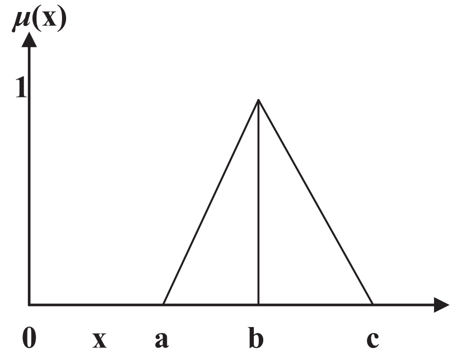

Figure (1) : Linear Fuzzy Real number

Let LFR = $\{\mu : R \to [0, 1] \mid \mu$ is a linear fuzzy real number$\}$. Each μ has a set of descriptive parameters. The base is defined as the triple (a, b, c) that occurs in the definition of a linear fuzzy real number. Thus one may write an element of LFR as $\mu = \mu(a, b, c)$[2].

2.1 The properties of Linear Fuzzy Real Numbers:

a. the linear fuzzy real numbers $\mu 1 = \mu(a1, b1, c1)$ and $\mu 2 = \mu(a2, b2, c2)$, $\mu 1 + \mu 2$ is defined by

$$\mu 1 + \mu 2 = \mu(a1 + a2, b1 + b2, c1 + c2).$$

b. A linear fuzzy real number $\mu(a, b, c)$ is defined to be positive if $a > 0$, negative if $c < 0$, and zero if $a \le 0$ and $c \ge 0$.
The following properties also hold:
1. If μ is positive , then $-\mu$ is negative;
2. If μ is negative, then $-\mu$ is positive;
3. If μ is zero, then $-\mu$ is also zero;
4. If $\mu 1$ and $\mu 2$ are positive, then so is $\mu 1 + \mu 2$;
5. If $\mu 1$ and $\mu 2$ are negative, then so is $\mu 1 + \mu 2$;
6. If $\mu 1$ and $\mu 2$ are zero, then so is $\mu 1 + \mu 2$;
7. For any μ, $\mu - \mu$ is zero.

c. Given the linear fuzzy real numbers $\mu 1 = \mu(a1, b1, c1)$ and $\mu 2 = \mu(a2, b2, c2)$,
$\mu 1 \cdot \mu 2$ is defined by

$\mu 1 \cdot \mu 2 = \mu(\min\{a1a2, a1c2, a2c1, c1c2\}, b1b2, \max\{a1a2, a1c2, a2c1, c1c2\})$.
Thus if $\mu i = \mu(ai, bi, ci)$ for $i = 1, 2, 3$, then $\mu 1 \cdot \mu 2 \cdot \mu 3 = \mu(\min\{a1a2a3, \cdot \cdot \cdot, c1c2c3\}, b1b2b3, \max\{a1a2a3, \cdot \cdot \cdot, c1c2c3\})$. Also, $\mu(a, b, c) \cdot \mu(1, 1, 1) = \mu(\min\{a, c\}, b, \max\{a, c\}) = \mu(a, b, c)$, i.e. $\mu \cdot$ _(1) = μ for all μ € *LFR*.

e. Given the linear fuzzy real numbers $\mu 1 = \mu(a1, b1, c1)$ and $\mu 2 = \mu(a2, b2, c2)$, $\mu 1/ \mu 2$ is defined by

$$\mu 1/ \mu 2 = \mu 1 . 1/ \mu 2$$

where $1/ \mu 2 = \mu(\min\{1/ a2, 1/ b2, 1/ c2\}, \text{median}\{1/ a2, 1/ b2, 1/ c2\}, \max\{1/ a2, 1/ b2, 1/ c2\})$ Note that for $\mu = \mu(a, b, c)$, if $0 < a \le b \le c$ then $1/ \mu = \mu(1/ a, 1/ b, 1/ c)$.

f. Given $\mu 1, \mu 2$ € LFR, $\mu 1 \le \mu 2$ provided that $a1 \le a2, b1 \le b2, c1 \le c2$. If €(0) $\le \mu(a, b, c)$, then $0 \le a \le b \le c$, hence μ is a non-negative linear fuzzyreal number[6].

3. Fuzzy Linear programming

The approach proposed here is based on an interaction with the decision maker, the implementer and the analyst in order to find a compromised satisfactory solution for a fuzzy linear programming (FLP) problem. In a decision process using FLP model, source resource variables may be fuzzy, instead of precisely given numbers as in crisp linear programming (CLP) model. For example, machine hours, labor force, material needed and so on in a manufacturing center, are always imprecise, because of incomplete information and uncertainty in various potential suppliers and environments.

A general model of crisp linear programming is formulated as :

$$Max\ z = c\ x$$
$$\text{Subject to}$$
$$Ax \geq or = b$$
$$x \gtrsim 0$$

where c and x are n dimensional vectors, b is an m dimensional vector, and A is m x n matrix. Since we are living in an uncertain environment, the coefficients of objective function (c), the technical coefficients of matrix (A) and the resource variables (b) are fuzzy. Therefore it can be represented by fuzzy numbers, and hence the problem can be solved by FLP approach.

The fuzzy linear programming problem is formulated as :

$$\text{Minimize } p^T \mu x$$
$$\text{subject to:}$$
$$M\mu x \geq or = \mu b,$$
$$\mu x \geq €(0).$$

where x is the vector of decision variables ; A , b *and* c are fuzzy quantities ; the operations of addition and multiplication by a real number of fuzzy quantities are defined by Zades.s extension principle (Zadeh, 1975) ; the inequality relation $\geq$ *or* = is given by a certain fuzzy relation and the objective function, z , is to be maximized in the sense of a given crisp LP problem[7,8,9,10].

4. Algorithm of fuzzy linear programming by using Two-phase

A fuzzy algorithm seeks to solve a problem through a series of logical operations. With a fuzzy algorithm we do not seek precision answers. Problems of the precision class are naturally restrictive and thus there arises a need for fuzzy algorithms applicable to fuzzy situations. The following algorithm evaluates each of the extreme points in *LFR/Z* and stores its *LFR* counterpart, thus our optimal solution has a fuzzy and a crisp version.

Schematic representation of the Two-phase algorithm may be summarized in the following steps.

Phase I.

a. Translate the technical specification of the problem into a fuzzy inequality and make a statement. Thus we should have a fuzzy objective and constraints, for example

$$\text{Minimize } p^T \mu x$$
$$\text{subject to:}$$
$$M\mu x \geq or = \mu b,$$
$$\mu x \geq €(0).$$

b. Convert the fuzzy inequality into a fuzzy equality by the addition of nonnegative slack variables. Modify the objective function to include the slack variables.

C. Modify the main body to consist of a “tri-matrix”, such that one matrix of triplets becomes three matrices of singlets. Hence we let $(\mu ij) = (A,B,C)$,
where $A = (aij)$, $B = (bij)$, and $C = (cij)$.

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d. Put the problem equation form, add the necessary artificial variable constraint of types (=) or (≥) to secure a starting basic solution, and find a basic solution of the resulting equations whether FLP I maximization or minimization always minimizes the sum of the artificial variables. If the minimum value of the sum is positive the FLP has no feasible solution which ends the process.

For both the maximization and the minimization problem be heaving variables is the basic variable associated with smallest nonnegative ratio and do the following.

e. Gauss-Jordan row operations.

phase1. Pivot row. Replace the heaving variable in the basic column with the entering variable, and New pivot row = Current pivot row ÷ pivot element. All other rows including z New row = (Current row) – (pivot column coefficient) x (New pivot row) F.

Do the following steps:

(1). Determine a starting basic feasible solution.

(2). Select variable; the last solution is optimal. Else go to (3).

(3). Select a leaving variable using the feasibility condition.

(4). Determine the new basic solution by using the Gauss-Jordan computation. Go to (2).

Phase II. Use the feasible solution from Phase 1 is a starting basic feasible solution for the original problem.

5. Examples of linear fuzzy programming

5.1 Example 1

Minimize $Z = 3\mu_{x1} + 2\,\mu_{x2} + 2\,\mu_{x3} + \mu_{x4}$

Subject to

$\mu(1, 2, 2).\ \mu_{x1} + \mu_{x2} = \mu(2, 3, 4)$

$\mu_{x2} + 4\,\mu_{x3} + \mu_{x4} \geq \mu(5/2, 5, 6)$

$\mu(1/2, 1, 1/2).\ \mu_{x1} + 3\,\mu_{x2} + \mu(1/2, 1, 1/2).\ \mu_{x4} \leq \mu(7/4, 6, 7)$ $\mu_i \geq 0$

inequality form with artificial variables and slack variable

$\mu(1, 2, 2).\ \mu_{x1} + \mu_{x2} + k_1 = \mu(2, 3, 4)$

$\mu_{x2} + 3\,\mu_{x3} + \mu_{x4} - \mu_{x5} + k_2 = \mu(5/2, 6, 8)$

$\mu(1/2, 1, 1/2).\ \mu_{x1} + 3\,\mu_{x2} + \mu(1/2, 1, 1/2).\ \mu_{x4} + \mu_{x6} = \mu(7/4, 5, 7)$

$k1 = \mu(2, 3, 4) - \mu(1, 2, 2).\ \mu_{x1} - \mu_{x2}$

$K2 = \mu(5/2, 6, 8) - \mu_{x2} - 3\,\mu_{x3} - \mu_{x4} + \mu_{x5}$

$K = K1 + K2$

Table (1) : the result of Phase 1

	basic	μ_{x1}	μ_{x2}	μ_{x3}	μ_{x4}	K1	K2	μ_{x5}	μ_{x6}	μ_b
A	k	1	2	3	1	0	0	0	-1	9/2
	K_1	1	1	0	0	1	0	0	0	2
	K2	0	1	3	1	0	1	-1	0	5/2
	μ_{x6}	1/2	3	0	1/2	0	0	0	1	7/4
B	k	1	2	3	1	0	0	0	-1	9
	K_1	1	1	0	0	1	0	0	0	3
	K_2	0	1	3**	1	0	1	-1	0	5
	μ_{x6}	1/2	3	0	1/2	0	0	0	1	5
C	k	1	2	3	1	0	0	0	-1	12
	K_1	1	1	0	0	1	0	0	0	4
	K_2	0	1	3	1	0	1	-1	0	8
	μ_{x6}	1/2	3	0	1/2	0	0	0	1	7

	basic	μ_{x1}	μ_{x2}	μ_{x3}	μ_{x4}	K1	K2	μ_{x5}	μ_{x6}	μ_b
A	K	1	1	0	0	0	-1	1	-1	13/6
	K1	1	1	0	0	1	0	0	0	2
	μ_{x3}	0	1/3	1	1/3	0	1/3	-1/3	0	5/6
	μ_{x6}	1/2	3	0	1/2	0	0	0	1	7/4
B	K	1	1	0	0	0	-1	1	-1	4
	K1	1	1	0	0	1	0	0	0	3
	μ_{x3}	0	1/3	1	1/3	0	1/3	-1/3	0	5/3
	μ_{x6}	1/2	3**	0	1/2	0	0	0	1	5
C	K	1	1	0	0	0	-1	1	-1	28/3
	K1	1	1	0	0	1	0	0	0	4
	μ_{x3}	0	1/	1	1/	0	1/	-	0	8/

			3		3		3	1/3		3
	μ_{x6}	1/2	3	0	1/2	0	0	0	1	7

	basic	μ_{x1}	μ_{x2}	μ_{x3}	μ_{x4}	K1	K2	μ_{x5}	μ_{x6}	μ_b
A	K	5/6	0	0	-1/6	0	-1	1	-7/6	19/12
	K1	5/6	0	0	-1/6	1	0	0	-1/6	17/12
	μ_{x3}	-1/18	0	1	5/18	0	1/3	-1/3	-1/18	23/36
	μ_{x2}	1/6	1	0	1/6	0	0	0	1/3	7/12
B	K	5/6	0	0	-1/6	0	-1	-1	-4/3	7/3
	K1	5/6 **	0	0	-1/6	1	0	0	-1/3	5/3
	μ_{x3}	-1/18	0	1	5/18	0	1/3	-1/3	-1/9	10/9
	μ_{x2}	1/6	1	0	1/6	0	0	0	1/3	5/3
C	K	5/6	0	0	-1/6	0	-1	1	-7/6	7
	K1	5/6	0	0	-1/6	1	0	0	-1/6	5/3
	μ_{x3}	-1/18	0	1	5/18	0	1/3	-1/3	-1/18	17/9
	μ_{x2}	1/6	1	0	1/6	0	0	0	1/6	7/3

	basic	μ_{x1}	μ_{x2}	μ_{x3}	μ_{x4}	K1	K2	μ_{x5}	μ_{x6}	μ_b
A	K	0	0	0	0	-1	-1	1	-1	2/12
	μ_{x1}	1	0	0	-1/5	6/5	0	0	-1/5	17/10
	μ_{x3}	0	0	1	4/3	1/15	1/3	-1/3	-1/15	11/15
	μ_{x2}	0	1	0	1/15	-1/5	0	0	1/15	3/10
B	K	0	0	0	0	-1	-1	-1	-1	2/3
	μ_{x1}	1	0	0	-1/5	6/5	0	0	-2/5	2
	μ_{x3}	0	0	1	1/2	1/15	1/3	-1/3	-1/5	11/9
	μ_{x2}	0	1	0	0	-1	-1	1	-1	4/3
C	K	0	0	0	5/12	-1	-1/4	1/4	-1	15/4
	μ_{x1}	1	0	0	-1/5	6/5	0	0	-1/5	2
	μ_{x3}	0	0	1	4/3	1/15	1/3	-1/3	-1/15	2
	μ_{x2}	0	1	0	1/5	-1/5	0	0	7/30	13/12

Phase 2:

After detecting are artificial columns, the original problem is written as

Minimize $Z = 3\mu_{x1} +_2 \mu_{x2} + 2\mu_{x3} + \mu_{x4}$

Subject to

From the solution the $\mu_{x4} = 0$

μ_{x1} -1/5 μ_{x4} = 17/10
μ_{x1} -1/5 μ_{x4} = 2
μ_{x1} -1/5 μ_{x4} = 2

the same thing for μ_{x2} and μ_{x3} = 0
Then
$\mu_{x1} = \mu(17/10, 2, 2)$, $\mu_{x2} = \mu(3/10, 4/3, 13/12)$, $\mu_{x3} = \mu(11/15, 11/9, 2)$

LFR/Z which optimal value are:

μ_{x1} = €(2): μ_{x2} = €(4/3): μ_{x3} = €(11/9)

The minimum of objective function is μ (25/6, 100/9, 73/6) in LFR and the crisp value is €(100/9) in LFR/Z.

5.2. Example 2

Maximize $Z = 5\mu_{x1} + 6\mu_{x2}$
Subject to
$\mu_{x1} + 3\mu_{x2} \geq \mu(2, 3, 5)$: $\mu_{x1} + \mu_{x2} - \mu_{x3} + k1 = \mu(4, 5, 6)$
$2\mu_{x1} - \mu_{x2} \geq \mu(4, 6, 8)$: $2\mu_{x1} + 2\mu_{x2} - \mu_{x4} + k2 = \mu(4, 7, 8)$

$\mu_{xi} \geq$ i= 1.2:

$k1 = \mu\,(2, 3,5) - \mu_{x1} - \mu_{x2} + \mu_{x3}$
$k2 = \mu\,(4, 6,8) - 2\mu_{x1} + 2\,\mu_{x2} + \mu_{x4}$

$k = -\,k1 - k2 = 3\mu_{x1} - \mu_{x2} - \mu_{x3} - \mu_{x4} - \mu\,(7, 11, 14)$

Table (2) : The result of Phase I

	basic	μ_{x1}	μ_{x2}	K1	K2	μ_{x3}	μ_{x4}	μ_b
A	k	-3	1	0	0	1	1	-8
	K1	1	1	1	0	-1	0	4
	K2	2	-1	0	1	0	-1	4
B	k	-3	-2	0	0	1	1	-12
	K1	1	3	1	0	-1	0	5
	K2	2**	-1	0	1	0	-1	7
C	k	-3	-2	0	0	1	1	-14
	K1	1	3	1	0	-1	0	6
	K2	2	-1	0	1	0	-1	8

	basic	μ_{x1}	μ_{x2}	K1	K2	μ_{x3}	μ_{x4}	μ_b
A	k	0	-1/2	0	3/2	1	-1/2	-2
	K1	0	3/2	1	-1/2	-1	1/2	2
	μ_{x1}	1	-1/2	0	1/2	0	-1/2	2
B	k	0	-1/2	0	3/2	1	-1/2	-5
	K1	0	3/2	1	-1/2	-1	1/2	3/2
	μ_{x1}	1	-1/2	0	1/2	0	-1/2	7/2
C	k	0	-1/2	0	3/2	1	-1/2	-2
	K1	0	3/2	1	-1/2	-1	1/2	2
	μ_{x1}	1	-1/2	0	1/2	0	-1/2	4

	Basic	μ_{x1}	μ_{x2}	K1	K2	μ_{x3}	μ_{x4}	μ_b
A	K	0	0	1/3	4/3	2/3	-2/3	-5/3
	μ_{x2}	0	1	2/3	-1/3	-2/3	1/3	2/3
	μ_{x1}	1	0	1/3	1/3	-1/3	-1/3	5/3
B	k	0	0	1/3	4/3	2/3	-2/3	11/2
	μ_{x2}	0	1	2/3	-1/3	-2/3	1/3	1
	μ_{x1}	1	0	1/3	1/3	-1/3	-1/3	4
C	k	0	-1/2	0	3/2	1	-1/2	-4/3
	μ_{x2}	0	1	2/3	-1/3	-2/3	1/1	4/3
	μ_{x1}	1	0	1/3	1/3	-1/3	-1/3	14/3

Phase II:
Maximize $Z = 5\,\mu_{x1} + 6\,\mu_{x2}$

Subject to
From the solution the $\mu_{x3}, \mu_{x4} = 0$
$\mu_{x1} - 1/3\,\mu_{x3} - 1/3\,\mu_{x4} = 5/3$
$\mu_{x1} - 1/3\,\mu_{x3} - 1/3\,\mu_{x4} = 4$
$\mu_{x1} - 1/3\,\mu_{x3} - 1/3\,\mu_{x4} = 14/3$

$\mu_{x1} = \mu\,(5/3, 4, 14/3)$ $\mu_{x2} = \mu\,(2/3, 1, 4/3)$

LFR/Z which optimal value are:

μ_{x1} = €(4): μ_{x2} = €(1): The maximum of objective function is μ (37/3, 26, 94/3) in LFR and the crisp value is €(26) in LFR/Z.

6. Conclusion

From our examples, There is a new method is proposed to find the fuzzy optimal solution of LFR problems with inequality constraints by representing all the parameters as triangular fuzzy numbers. it is clear that we see these is relation (≥ or =) we use tow phase method technique to solve problem of LFR, which can find a crisp optimum by determine in the middle of $\mu(a, b, c) \rightarrow €(b)$. Also the method outlined operation a fuzzy solution in the form of an *LFR* expression. A fuzzy value or in an interval form by suppressing the middle entry. Therefore, $\mu(a, b, c) \rightarrow [a, c]$ is then the relevant projection. Thus the LFR-method can be appear as a fuzzy hybrid/extension, which preserves aspects of both methods simultaneously. These method approach to the real world problems, especially in situations where it is already known that "crisp optima" in the purest sense do not exist, but where $\mu(a, b, c) \rightarrow €(b)$ produces a "crisp good choice" for an optimum.

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