Stable Robust Adaptive Control of Induction Motors with Unknown Parameters

Ibrahim Fahad Jasim
Electrical and Electronic Engineering Department
College of Engineering, University of Kerbala
Kerbala, Iraq
E-mail: ibrahim.jasim@ieee.org

***Abstract*—This paper presents a new strategy for controlling induction motors with unknown parameters. Using a simple linearized model of induction motors, we design robust adaptive controllers and unknown parameters update laws. The control design and parameters estimators are proved to have global stable performance against sudden load variations. All closed loop signals are guaranteed to be bounded. Simulations are performed to show the efficacy of the suggested scheme.**

I. Introduction

Induction motors are very popular in industrial and domestic applications. So, controlling this type of machines is a very important aspect to both control and power practitioners. Early control schemes implementing decoupled control of machine flux and torque are exemplified by Field-Oriented Control (FOC) as proposed by Blaschke [1] and Leonhard [2]. The main idea of the FOC is to resolve the stator current into two parts; one controls the torque production and the other controls the flux. Thus the torque and flux would be controlled independently. However complete decoupling was not yet achieved. In [3-5], feedback linearization was successfully used to obtain a complete decoupling of torque and flux control, opening up a new direction of controlling the induction motors that torque (with position and velocity) and flux can be controlled separately.

The advent of control theory caused a significant impact on the induction motors control. Both adaptive control and sliding mode concept was used to control the induction motors with known parameters and promising results were obtained (see [6] and references therein). Moreover, adaptive fuzzy sliding mode control was successfully used to have a desirable position tracking error for induction motors [7] with unknown parameters. However, the technique described in [7] has the following drawbacks:

1. The fuzzy approximator fitted to operate within a restricted region of operation.
2. Only the subsystem corresponds to the torque is controlled. The flux remains uncontrolled.

In this paper, we address the problem of controlling the induction motor with unknown parameters. Both decoupled subsystems of the induction motor are taken into account. The angular position, speed, and flux are controlled to follow a prescribed reference signals.

The rest of the paper is organized as follows. In section 2, we present the control problem of the induction motor to be considered throughout this paper. The robust adaptive controllers and unknown parameters estimators are given in section3. Section 4 illustrates the simulation results and section 5 summarizes the concluding remarks.

II. Problem Statement

The mathematical model of a field oriented current command induction motor can be written as [4,8]:

$$\frac{d\theta}{dt} = \omega \tag{1.a}$$

$$\frac{d\omega}{dt} = \mu\psi_d i_{qr} - \frac{f}{J}\omega - \frac{T_L}{J} \tag{1.b}$$

$$\frac{d\psi_d}{dt} = -\eta\psi_d + \eta M i_{dr} \tag{1.c}$$

$$\frac{d\rho}{dt} = n_p\omega + \frac{\eta M i_q}{\psi_d} \tag{1.d}$$

With

$$\eta \triangleq \frac{R_R}{L_R},\ \mu \triangleq \frac{n_p M}{J L_R}$$

Where θ is the shaft angular displacement, ω is the angular speed, J is the combined shaft and load ineria, n_p is the number of pole pair, R_R and L_R are the rotor resistance and inductance respectively, M is the motor mutual inductance, ψ_d is the flux magnitude, ρ is the flux angle, i_{qr} is the quadrature current component, and i_{dr} is the direct axis current component.

Normally, ρ is used to estimated the current and (or) speed, and since it is an angle, then its value is always limited between 0^o and 360^o. So, it is always bounded and it rarely to be a control objective.

The parameters f,J, η, and M are assumed to be unknown. All states are assumed to be available for measurement.

Now, let:

$u_1 = i_{qr}\mu\psi_d, u_2 = i_{dr},\ a_1 = \frac{f}{J}, a_2 = \frac{1}{J}, a_3 = \eta, a_4 = \eta M$

Then the model given in (1) can be rewritten as:

$$\frac{d\theta}{dt} = \omega \quad (2.a)$$

$$\frac{d\omega}{dt} = -a_1\omega - a_2T_L + u_1 \quad (2.b)$$

$$\frac{d\psi_d}{dt} = -a_3\psi_d + a_4u_2 \quad (2.c)$$

The objective of the paper is to derive robust adaptive control laws (for $u_1$and u_2) and parameters update laws for a_1, a_2, a_3, and a_4 such that θ, ω, and $\psi_d \rightarrow \theta_{ref}$, ω_{ref} , and ψ_{dref} respectively as $t \rightarrow \infty$ provided that all closed loop signals are bounded. However, the following assumptions are needed to be satisfied:

A1. All states θ, ω, and $\psi_d$are assumed to be available for measurement.

A2. All reference signalsθ_{ref},ω_{ref}, and $\psi_{dref}$are assumed to be bounded.

A3. The parameters a_1, a_2, a_3 and $a_4$are bounded.

III. ROBUST ADAPTIVE CONTROL DESIGN

Before we present the main theorem of this paper, we define several concepts. Let:

$$e_1 = \theta - \theta_{ref} \quad (3.a)$$

$$e_2 = \omega - \omega_{ref} \quad (3.b)$$

$$e_3 = \psi_d - \psi_{dref} \quad (3.c)$$

It is clear that the dynamic model given in (2) consists of two decoupled subsystems, say (2.a, 2.b) and (2.c). So, we shall define two sliding surfaces, say $s_1$and s_2, as shown below:

$$s_1 = \left(\frac{d}{dt} + \lambda_1\right) e_1 \qquad \lambda_1 > 0 \quad (4.a)$$

$$s_2 = \left(\frac{d}{dt} + \lambda_2\right)^0 e_3 \qquad \lambda_2 > 0$$

$$\therefore\ s_2 = e_3 \quad (4.b)$$

Taking the time derivative for (4.a) and (4.b), we obtain:

$$\dot{s_1} = \ddot{e_1} + \lambda_1\dot{e}$$

$$\therefore \dot{s_1} = \ddot{\theta} - \ddot{\theta}_{ref} + \lambda_1(\dot{\theta} - \dot{\theta}_{ref}) \quad (5.a)$$

$$\dot{s}_2 = \dot{e}_3$$

$$\therefore \dot{s}_2 = \dot{\psi}_d - \dot{\psi}_{dref} \quad (5.b)$$

Note: It was shown that the filtered errors given in (4.a) and (4.b) has the following properties: (i) the equations $s_1(t) = 0$ and $s_2(t) = 0$ define time-varying hyperplanes in R^2 and R, on which the tracking errors e_1, e_2, and e_3 decays ecponetially to zero. (ii) if $e(0) = 0$ and $|s(t)| \leq \varepsilon$ with constant ε, then $e(t) \in \Omega_\varepsilon = \left\{\frac{e(t)}{e_i(t)} \leq 2^{i-1}\lambda^{i-n}\varepsilon, i = 1,2 \text{ for } s_1 \text{ and } i = 1 \text{ for } s_2\right\}$ for all $t \geq 0$. (iii) if $e(0) \neq 0$ and $|s(t)| \leq \varepsilon$, then $e(t)$ will converge to Ω_ε within a time constant $\frac{(n-1)}{\lambda}$ (see [9,10]).

Define the parameters errors to be:

$$\tilde{a}_1 = \hat{a}_1 - a_1^* \quad (6.a)$$

$$\tilde{a}_2 = \hat{a}_2 - a_2^* \quad (6.b)$$

$$\tilde{a}_3 = \hat{a}_3 - a_3^* \quad (6.c)$$

$$\tilde{a}_4 = \hat{a}_4 - a_4^* \quad (6.d)$$

Define also the modified filtered error:

$$s_{\varepsilon1} = s_1 - \varepsilon_1 \text{sat}\left(\frac{s_1}{\varepsilon_1}\right) \quad (7.a)$$

$$s_{\varepsilon2} = s_2 - \varepsilon_2 \text{sat}\left(\frac{s_2}{\varepsilon_2}\right) \quad (7.b)$$

Theorem: For the induction motor given in (2) satisfying **A1**, **A2** and **A3**, the controllers given in (8.a and 8.b) along with the parameters update laws (8.c, 8.d, 8.e, and 8.f) can guarantee global system stability and enhanced tracking performance.

$$u_1 = -k_{d1}s_1 + \hat{a}_1\omega + \hat{a}_2T_L - \lambda_1(\omega - \omega_{ref}) \quad (8.a)$$

$$u_2 = \frac{1}{\hat{a}_4}(-k_{d2}s_2 + \hat{a}_3\psi_d + \dot{\psi}_{dref}) \quad (8.b)$$

$$\dot{\hat{a}}_1 = -\gamma_1\omega s_{\varepsilon1} \quad (8.c)$$

$$\dot{\hat{a}}_2 = -\gamma_2\hat{a}s_{\varepsilon1} \quad (8.d)$$

$$\dot{\hat{a}}_3 = -\gamma_3\psi_d s_{\varepsilon2} \quad (8.e)$$

$$\dot{\hat{a}}_4 = -\frac{\gamma_4 s_{\varepsilon 2}}{\hat{a}_4}\left(k_{d2}s_2 - \psi_d - \dot{\psi}_{dref}\right) \quad (8.f)$$

Proof: Consider the Lyapunov candidate:

$$V = \frac{1}{2}\left[s_{\varepsilon 1}^2 + s_{\varepsilon 2}^2 + \frac{\tilde{a}_1^2}{\gamma_1} + \frac{\tilde{a}_2^2}{\gamma_2} + \frac{\tilde{a}_3^2}{\gamma_3} + \frac{\tilde{a}_4^2}{\gamma_4}\right] \quad (9)$$

Taking the time derivative of (9), we obtain:

$$\dot{V} = s_{\varepsilon 1}.\dot{s}_{\varepsilon 1} + s_{\varepsilon 2}.\dot{s}_{\varepsilon 2} + \frac{1}{\gamma_1}\tilde{a}_1.\dot{\hat{a}}_1 + \frac{1}{\gamma_2}\tilde{a}_2.\dot{\hat{a}}_2 + \frac{1}{\gamma_3}\tilde{a}_3.\dot{\hat{a}}_3 + \frac{1}{\gamma_4}\tilde{a}_4.\dot{\hat{a}}_4 \quad (10)$$

From (7.a and 7.b), we can easily conclude that:

$$\dot{s}_{\varepsilon 1} = \dot{s}_1$$

$$\dot{s}_{\varepsilon 2} = \dot{s}_2$$

Using (5.a and 5.b), then (10) can be rewritten as:

$$\dot{V} = s_{\varepsilon 1}.\left(\ddot{\theta} - \ddot{\theta}_{ref} + \lambda_1\left(\dot{\theta} - \dot{\theta}_{ref}\right)\right) + s_{\varepsilon 2}.\left(\dot{\psi}_d - \dot{\psi}_{dref}\right)$$

$$+\frac{1}{\gamma_1}\tilde{a}_1.\dot{\hat{a}}_1 + \frac{1}{\gamma_2}\tilde{a}_2.\dot{\hat{a}}_2 + \frac{1}{\gamma_3}\tilde{a}_3.\dot{\hat{a}}_3 + \frac{1}{\gamma_4}\tilde{a}_4.\dot{\hat{a}}_4 \quad (11)$$

Substituting (2.a, 2.b, and 2.c) into (11), then we obtain:

$$\dot{V} = s_{\varepsilon 1}.\left(-a_1\omega - \ddot{a}_2 T_L + u_1 - \ddot{\theta}_{ref} + \lambda_1(\dot{\omega} - \dot{\omega}_{ref})\right)$$

$$+s_{\varepsilon 2}.\left(-a_3\psi_d + a_4 u_2 - \dot{\psi}_{dref}\right) + \frac{1}{\gamma_1}\tilde{a}_1.\dot{\hat{a}}_1 + \frac{1}{\gamma_2}\tilde{a}_2.\dot{\hat{a}}_2$$

$$+\frac{1}{\gamma_3}\tilde{a}_3.\dot{\hat{a}}_3 + \frac{1}{\gamma_4}\tilde{a}_4.\dot{\hat{a}}_4 \quad (12)$$

Using the controllers defined in (8.a and 8.b), then we obtain:

$$\dot{V} = s_{\varepsilon 1}.(-a_1\omega - a_2 T_L + (-k_{d1}s_1 + \hat{a}_1\omega + \hat{a}_2 T_L \quad -\lambda_1(\omega - \omega_{ref})) - \ddot{\theta}_{ref} + \lambda_1(\dot{\omega} - \dot{\omega}_{ref})) +$$
$$s_{\varepsilon 2}.\left(-a_3\psi_d + a_4\left(\frac{1}{\hat{a}_4}(-k_{d2}s_2 + \hat{a}_3\psi_d + \dot{\psi}_{dref})\right) - \dot{\psi}_{dref}\right) +$$
$$\frac{1}{\gamma_1}\tilde{a}_1.\dot{\hat{a}}_1 + \frac{1}{\gamma_2}\tilde{a}_2.\dot{\hat{a}}_2 + \frac{1}{\gamma_3}\tilde{a}_3.\dot{\hat{a}}_3 + \frac{1}{\gamma_4}\tilde{a}_4.\dot{\hat{a}}_4 \quad (13)$$

After several simple mathematical manipulations for (13), we can obtain:

$$\dot{V} = -k_{d1}.s_{\varepsilon 1}.s_1 - k_{d2}.s_{\varepsilon 2}.s_2 + \tilde{a}_1\left(s_{\varepsilon 1}.\omega + \frac{1}{\gamma_1}\dot{\hat{a}}_1\right)$$

$$+\tilde{a}_2\left(s_{\varepsilon 1}.T_L.\hat{a}_4 + \frac{1}{\gamma_2}\dot{\hat{a}}_2\right) + \tilde{a}_3\left(s_{\varepsilon 2}.\psi_d + \frac{1}{\gamma_3}\dot{\hat{a}}_3\right)$$

$$+\tilde{a}_4\left(\frac{k_{d2}.s_2 - \psi_d - \dot{\psi}_{dref}}{\hat{a}_4} + \frac{1}{\gamma_4}\dot{\hat{a}}_4\right) \quad (14)$$

Using the parameters updates laws given in (8.c, 8.d, 8.e, and 8.f) and the relation given in (7.a and 7.b), we obtain:

$$\dot{V} = -k_{d1}.s_{\varepsilon 1}.\left(s_{\varepsilon 1} + \varepsilon_1.\mathrm{sat}\left(\frac{s_1}{\varepsilon_1}\right)\right) - k_{d2}.s_{\varepsilon 2}.\Big(s_{\varepsilon 2}$$

$$+\varepsilon_2.\mathrm{sat}\left(\frac{s_2}{\varepsilon_2}\right)\Big) \quad (15)$$

$$\dot{V} = -k_{d1}.s_{\varepsilon 1}^2 - k_{d1}.\varepsilon_1.s_{\varepsilon 1}.\mathrm{sat}\left(\frac{s_1}{\varepsilon_1}\right) - k_{d2}.s_{\varepsilon 2}^2 - k_{d2}.\varepsilon_2.s_{\varepsilon 2}.\mathrm{sat}\left(\frac{s_2}{\varepsilon_2}\right) \quad (16)$$

Then $\dot{V}$ can satisfy inequality below:

$$\dot{V} \le -k_{d1}.s_{\varepsilon 1}^2 - k_{d1}.\varepsilon_1.|s_{\varepsilon 1}| - k_{d2}.s_{\varepsilon 2}^2 - k_{d2}.\varepsilon_2.|s_{\varepsilon 2}| \quad (17)$$

From (17), it is clear that $s_{\varepsilon 1,2} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4 \in \mathcal{L}_\infty$. Since a_1, a_2, a_3 and a_4 are bounded and $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4 \in \mathcal{L}_\infty$ then $\hat{a}_1, \hat{a}_2, \hat{a}_3$ and $\hat{a}_4$ are also bounded. Using (5.a and 5.b), then we can easily conclude that $\dot{s}_{1,2} \in \mathcal{L}_\infty$ which implies that $\dot{s}_{\varepsilon 1,2} \in \mathcal{L}_\infty$. Since we have $s_{\varepsilon 1,2} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\dot{s}_{\varepsilon 1,2} \in \mathcal{L}_\infty$, then $s_{\varepsilon 1,2} \to 0$ as $t \to \infty$ according to Barbalat's lemma. This would force $\tilde{\theta}, \tilde{\omega}$, and $\tilde{\psi}_d$ for converging to $\Omega_{\varepsilon 1,2}$. ■

IV. Simulation Results

Simulations were carried out for an induction machine with the following parameters:

Connection type is Y, voltage rating is 380 V, current rating is 5 A, number of phase 3 ph, rated power is 2.2 kW, frequency is 50 Hz, rated speed is 1430 rpm, R_r is 1.7 Ω, , L_r is 0.34 H , R_s is 3.5 Ω, , L_s is 0.31 H, L_m is 0.29 H, J is $4.78 \times 10^{-3} Nm/s^2$, and f is $5.34 \times 10^{-3} Nms/rad$.

The position, velocity and flux reference signals are:

$$\theta_{ref} = \sin(0.4\pi t),\ \omega_{ref} = 0.4\pi\cos(0.4\pi t)$$

$$\psi_{dref} = 0.5\sin(0.4\pi t)$$

The load torque is assumed to be varying in a square wave fashion of 0.2 Hz frequency and peaks of +1 and -1.

Figures 1, 2, and 3 show the position, velocity, and flux tracking performance. It is clear that excellent tracking performance was obtained for all three states, say θ, ω, and ψ_d . As illustrated through the section 3and the theorem therein, all states errors would converge asymptotically to bounded region, say $\Omega_{\varepsilon 1,2}$. The boundary of this region can be specified by the designer through choosing appropriate values for $\varepsilon_{1,2}$and $\lambda_{1,2}$which is done through trade-off. Smaller values of $\lambda_{1,2}$ would cause the regions $\Omega_{\varepsilon 1,2}$ to be smaller, however the convergence time would be increased. Similarly, smaller values of $\varepsilon_{1,2}$ would cause smaller regions of $\Omega_{\varepsilon 1,2}$, however chattering would be caused spurring high frequencies that may constitute a threat to the overall system stability [9,10]. In our design we used the values of ε_1 and ε_2 to be 0.01. For λ_1 and λ_2, we took the values of 1 and 0.1 respectively.

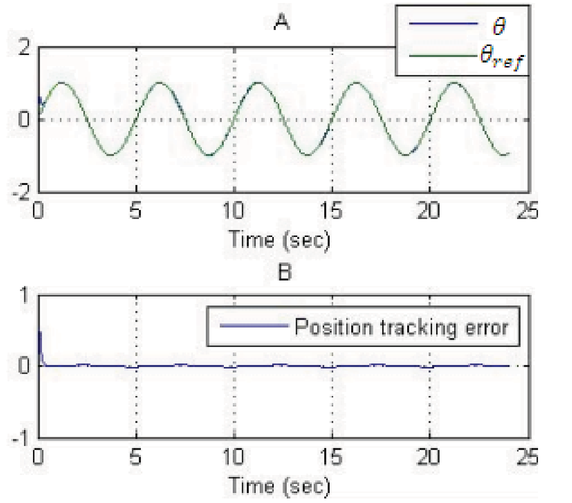

Figure 1 A. Reference and actual angular position (in rad) B. Position tracking error (in rad)

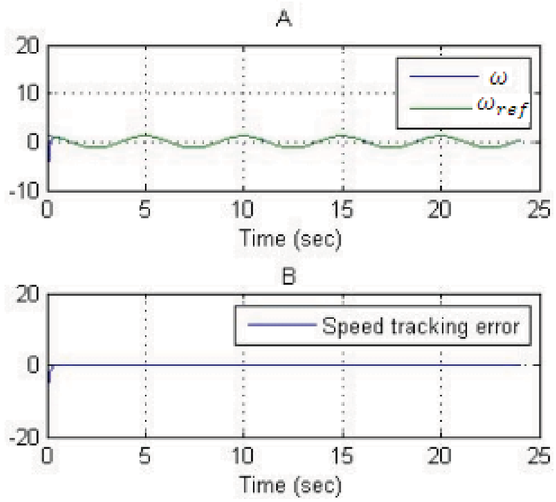

Figure 2 A. Reference and actual angular speed (in rad/sec) B. Speed tracking error (in rad/sec)

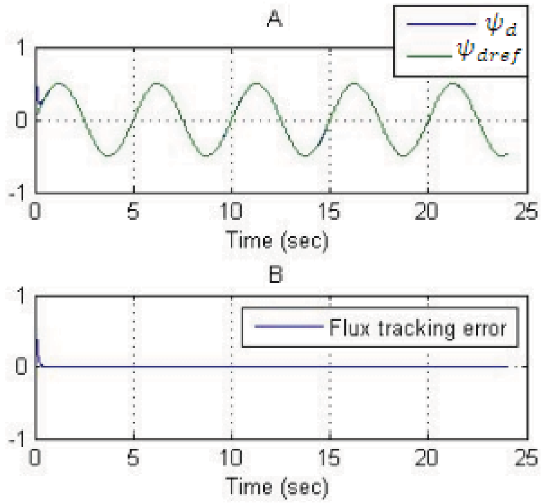

Figure 3 A. Reference and actual flux (in web) B. Flux tracking error (in web)

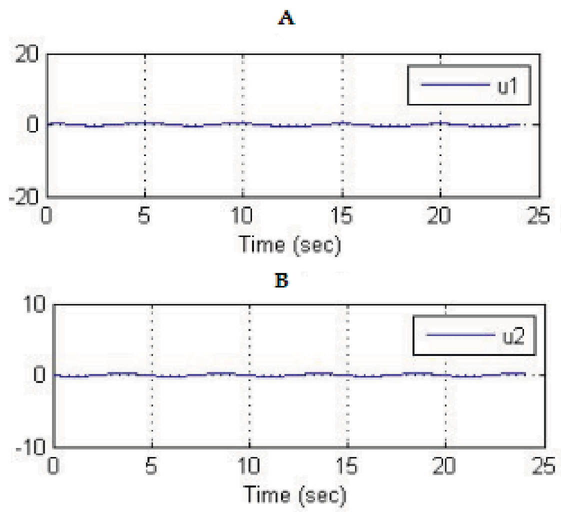

Figure 4 A. Torque control action u_1 B. Flux control action u_2

Both control actions are given in Figure 4. Both control actions are functions of the signals that were proved to be bounded. So, both of the control actions would be bounded (See Figure 3 A and B). The parameters $\hat{a}_1$,$\hat{a}_2$,$\hat{a}_3$, and $\hat{a}_4$ ere also proved to be bounded and as per checking Figure 5, it is clear that all the parameters estimated are bounded as proved in the paper main theorem.

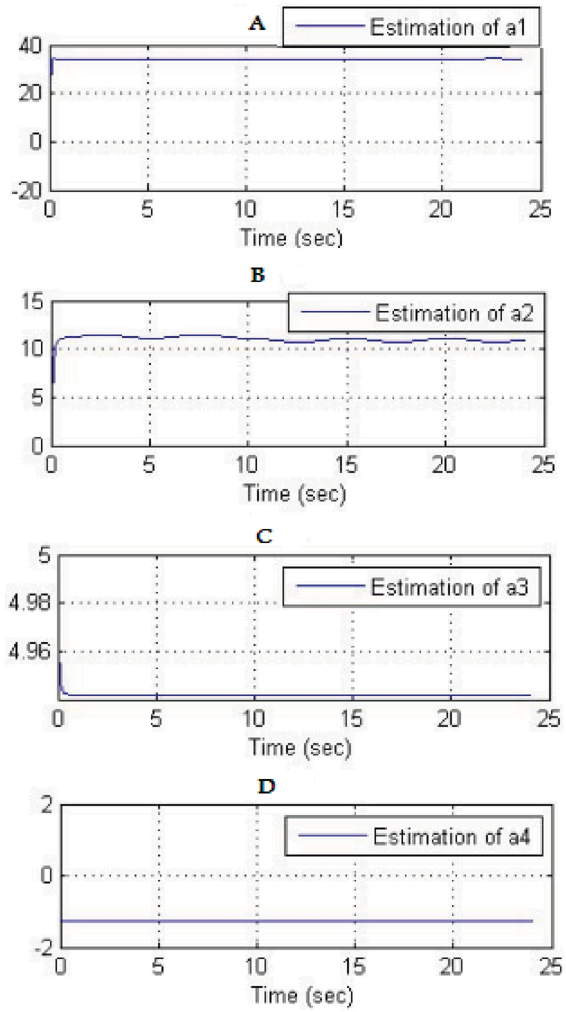

Figure 5 A. Estimation of a_1 B. Estimation of a_2 C. Estimation of a_3 D. Estimation of 4

V. CONCLUSION

The control problem of induction motors with unknown parameters was addressed. Robust adaptive control laws were derived for both flux and torque dynamics. Estimators for the unknown parameters were also suggested. The suggested scheme was shown to have global stable performance with all closed loop signals guaranteed to be bounded. However, the control actions derived are of high initial values that may break the current constraints. So, future works should focus on deriving new control schemes that keeps the control actions to be within a prescribed bound in order to keep the currents constraint valid.

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